What is Reliability?

Reliability is associated with unexpected failures of products or services, and understanding why these failures occur is key to improving reliability. The main reasons why failures occur include:

- The product is not fit for purpose, or more specifically, the design is inherently incapable.
- The item may be overstressed in some way.
- Failures can be caused by wear-out.
- Failures might be caused by vibration.

Other Potential Causes of Failures:

- Wrong specifications may cause failures.
- Misuse of the item may cause failure.
- Items are designed for a specific operating environment, and if they are used outside this environment, failure can occur.

Objectives of Reliability Engineering:

It is clear that to ensure good reliability, the causes of failure need to be identified and eliminated. The objectives of reliability engineering are:

- To apply engineering knowledge to prevent or reduce the likelihood or frequency of failures.
- To identify and correct the causes of failures that do occur.
- To determine ways of coping with failures that do occur.
- To apply methods of estimating the likely reliability of new designs, and for analyzing reliability data.

Definition of Reliability:

Reliability describes the ability of a system or component to function under stated conditions for a specified period of time.

Why is Reliability Important?

- Reputation
- Customer Satisfaction
- Warranty Costs
- Cost Analysis
- Customer Requirements
- Competitive Advantage

Here is the proofread version of the additional lecture content:

Other Measures of Reliability

Availability is used for repairable systems.

- It is the probability that the system is operational at any random time ttt.
- It can also be specified as a proportion of time that the system is available for use in a given interval (0,T)(0, T)(0,T).

Reliability Indices

Failure Rate (λ): A reliability index that represents the rate at which your product fails.

- **Mean Time To Failure (MTTF):** The reliability index for *non-repairable units* represents the mean time to failure.
- **Mean Time Between Failures (MTBF):** The reliability index for *repairable units* represents the mean time between failures.

Reliability Indices Formulas

- Failure Rate (λ):

$$\lambda = \frac{\text{Number of Failures}}{\text{Operating Time (Cycles)}}$$

  This represents failures per hour.

- Mean Time To Failure (MTTF):

$$MTTF = \frac{\text{Operating Time (Cycles)}}{\text{Number of Failures}}$$

- Mean Time Between Failures (MTBF):

$$MTBF = \frac{\text{Operating Time (Cycles)}}{\text{Number of Failures}}$$

Reliability Functions

- Reliability: $R(t) = e^{-\lambda t}$
- Alternatively: $R(t) = e^{-\left(\frac{t}{\theta}\right)}$ where λ is the failure rate, and θ is the mean time to failure.

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The Bathtub Curve

The **bathtub curve** is a reliability tool that is used to model the reliability of a unit or system over its entire life.

- **Early-Failure Period**
- **Useful-Life Period**
- **Wear-Out Period**

Bathtub Curve (Life-Cycle Curve and Probability Distributions in Modeling Reliability)

The bathtub curve is widely used in **reliability engineering**. It describes a particular form of the **hazard function** which comprises three parts:

1. **Decreasing failure rate** (known as early failures).
2. **Constant failure rate** (known as random failures).
3. **Increasing failure rate** (known as wear-out failures).

The name is derived from the cross-sectional shape of a bathtub: steep sides and a flat bottom.

The bathtub curve is generated by mapping:

- The rate of early "infant mortality" failures when first introduced.
- The rate of random failures with a constant failure rate during its "useful life."
- The rate of "wear-out" failures as the product exceeds its design lifetime.

In less technical terms:

- **Early life of a product:** The failure rate is high but rapidly decreases as defective products are identified and discarded, and early sources of potential failure such as handling and installation errors are overcome.
- **Mid-life of a product:** For consumer products, the failure rate is low and constant.
- **Late life of a product:** The failure rate increases as age and wear take their toll on the product. Many electronic consumer product life cycles exhibit the bathtub curve strongly.

General Observations on Failures

- Some components have well-defined failures, while others do not.
  - **Initial Failure:** In the beginning, when the item or component is installed, it fails with high frequency, known as initial failure or infant mortality. These are generally due to manufacturing defects. The failure rate is high at the initial stages but gradually decreases and stabilizes over time.
  - **Stable Failures:** These are random failures that occur over a long period and are characterized by a constant number of failures per unit of time.
  - **Wear-Out Failures:** Due to wear and tear with usage, the item gradually deteriorates, and the frequency of failures increases. At this stage, the failure rate becomes high due to deterioration, known as wear-out failures.

Reliability @ Different MTBFs

We've tested 20 units and found that our MTBF is 2,996 hours. What is the reliability of our product after 1,200 hours of operation?

Formula for Reliability:

$$R(t) = e^{-\lambda t} = e^{-\frac{1}{\theta}(t)}$$

Where:

- $\lambda = \frac{1}{\theta}$ (Failure Rate)
- $R(1,200) = e^{-\frac{1,200}{2,996}} \approx 0.6699$

The probability that our product will successfully perform past the 1,200-hour mark is approximately **66%**.

Alternative Interpretation:

66% of the population of units can be expected to surpass the 1,200-hour mark.

Graph Explanation:

The graph shows **Reliability** as a function of **Time**, with the MTBF values (5,000, 2,500, and 500 hours) represented by different curves. The reliability of the product decreases as the number of operating hours increases. In this case, for an MTBF of 2,996 hours, the reliability after 1,200 hours is approximately **66%**, as shown in the graph.

Example:

Problem:
What is the highest failure rate for a product if it is to have a probability of survival (i.e., successful operation) of 95% at 4000 hours? Assume that the time to failure follows an exponential distribution.

Solution:

The reliability at 4000 hours is 0.95. If the constant failure rate is given by λ, we have:

$$R(t) = e^{-\lambda t}$$

Given $R(t) = 0.95$ at 4000 hours:

$$0.95 = e^{-\lambda(4000)}$$

Solving for λ:

$$\lambda = 0.0000128 \text{ per hour} = \frac{12.8}{10^6} \text{ hours}$$

Thus, the highest failure rate is 12.8×10^6 hours for a reliability of 0.95 at 4000 hours.

The Weibull Distribution

The Weibull distribution was discovered by **Waloddi Weibull** and is the most versatile distribution in Reliability Engineering because of its ability to model a variety of distributions.

Weibull Distribution Phases:

1. **Early-Failure Period**
2. **Useful-Life Period**
3. **Wear-Out Period**

Reliability Function for Weibull Distribution

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

Where β (Beta) is the **Weibull Shape Parameter**.

Weibull Distribution Graphs:

- For different values of β (such as $\beta = 0.85, \beta = 1, \beta = 2, \beta = 3.5$), the graph demonstrates the varying failure rates across different life stages of the product.

Reliability Formula:

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

Where:

- β (Beta) = the Weibull Shape Parameter.
- θ (Theta) = the Weibull Scale Parameter.

The Shape (Slope) Parameter:

- When $\beta < 1$, the Weibull distribution represents a system with a **decreasing failure rate**.
- When $\beta = 1$, the Weibull distribution is approximately equal to the **exponential distribution**.
- When $\beta > 1$, the Weibull distribution represents a system with an **increasing failure rate**.

Weibull Reliability Example:

You've collected data on a component and calculated that your product fits a Weibull distribution with a **slope of 2**.
Your data also indicates that your **scale parameter** is equal to **8,000 hours**.

Question:
What is the reliability of your system at 5,000 hours?

Given:

- $\beta = 2$
- $\theta = 8,000 \text{ hours}$
- $t = 5,000 \text{ hours}$

Solution:

$$R(5,000) = e^{-\left(\frac{5,000}{8,000}\right)^{2}}$$

$$R(5,000) \approx 0.6766 \text{ or } 67\% \text{ Reliability}$$

Exponential Distribution

The exponential model, with only one unknown parameter, is the simplest of all life distribution models. The key equations for the exponential distribution are shown below:

- PDF (Probability Density Function):

$$f(t, \lambda) = \lambda e^{-\lambda t}$$

- CDF (Cumulative Distribution Function):

$$F(t) = 1 - e^{-\lambda t}$$

- Reliability:

$$R(t) = e^{-\lambda t}$$

- Failure Rate:

$$h(t) = \lambda$$

- Mean:

$$\frac{1}{\lambda}$$

- Median:

$$\frac{\ln 2}{\lambda} \approx \frac{0.693}{\lambda}$$

- Variance:

$$\frac{1}{\lambda^2}$$

Note that the failure rate reduces to the constant λ for any time. The exponential distribution is the only distribution to have a constant failure rate. Another name for the exponential mean is the **Mean Time To Fail (MTTF)**, and we have:

$$MTTF = \frac{1}{\lambda}$$